D-003-001513 Seat No. ______

B. Sc. (Sem. V) (CBCS) Examination

March - 2022

Mathematics : BSMT-501 (A)

(Mathematical Analysis & Group Theory)

Faculty Code : 003
Subject Code : 001513

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : All questions are compulsory.

1 Answer the following questions in short : **20**

(1) Define closer set.
(2) Give an example of a subset of metric space R which is not open and closed.
(3) Define interior point
(4) Define neighborhood
(5) Find the limit points of subset E=(1, 2) of metric space R.
(6) Define Upper Riemann integration
(7) If $f:[1,3]\rightarrow R, f(x)=\frac{1}{x}$ and P={1, 2, 3} then find U(P,f)
(8) Define partition of an interval
(9) Define Riemann sum
(10) Define Riemann integration
(11) Define subgroup
(12) Find the order of each element of group $(Z_4,+_4)$
(13) Define Abelian group
(14) If f = (1, 3, 4, 5, 2) then find O(f), where $f\in S_6$.
(15) Define Prmutation
(16) Define Transposition
(17) Find the order of cyclic subgroup generated by 25 of group Z_{60}
(18) Define index of subgroup H in group G
(19) Define normal sugroup
(20) Is $(Z_6,+_6)$ and $(Z_8,+_8)$ are isomorphic ? Give the reason.

2 (a) Answer any three : **6**

(1) Define matric space and discrete metric space.

(2) If (X, d) is a metric space and $A, B \subset X$ and $A \subset B$ and $A^{'} \subset B^{'}$.

(3) Obtain border set of the subset (1, 2) of metric space R.

(4) Determine whether set $\{x \in R / x^2 - 1 = 0\}$ is open or closed set.

(5) If f is continuous on [a, b] then for some $c \in [a, b]$ then show that $\int_a^b f(x)dx = f(c)\,(b-a)$.

(6) Evaluate : $\lim_{n\to\infty} n\sum_{r=1}^{n}\frac{1}{n^2+r^2}$.

(b) Answer any **three** : **9**

(1) If function f is constant on [a, b] then prove that f is Riemann integrable on [a, b].

(2) If f is increasing function on [a, b] then prove that f is R-integrable.

(3) If (X, d) is a metric space and $A \subset X$ then prove that A is closed $\Leftrightarrow A = \overline{A}$.

(4) Prove that the arbitrary intersection of closed sets of metric space is closed set.

(5) State and prove fundamental theorem of Riemann integration.

(6) If f is bounded function defined on [a, b] and M, m are lub and glb of f in [a, b] respectively.
Then there exists a partition P of [a, b] such that
$m(b-a) \le L\,(P, f) \le U\,(P, f) \le M(b-a)$

(c) Answer any two : **10**

(1) Prove that derived set of any subset of a metric space is a closed set.

(2) Prove that $\frac{1}{4}$ is in cantor set.

(3) Prove that $\frac{\pi^2}{6} \le \int_0^{\pi}\frac{x}{2+\cos x}\,dx \le \frac{x^2}{2}$

(4) Evaluate : $\lim_{n\to\infty}\left[\frac{1}{\sqrt{4n^2-1^2}}+\frac{1}{\sqrt{4n^2-2^2}}+\ldots\ldots\ldots+\frac{1}{\sqrt{3n^2}}\right]$

(5) State and prove necessary and sufficient condition for a bounded function to be R-integrable.

3 (a) Answer any three : **6**

(1) If $\sigma = (1\,2\,3\,4), \sigma \in S_6$ then find $O(\sigma)$.

(2) If $f : R \to R$ is defined as $f(x) = x + 1$ then check whether f is a permutation or not.

(3) If $f = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ then find fg.

(4) Show that (Z, +) is a group.

(5) Prove that every element of a finite group is of finite order.

(6) Check whehter (Z, +) is cyclic group or not.

(b) Answer any three : **9**

(1) If (G, *) is a group and $a, b \in G$. Then prove that a*x=b and y*a=b have unique solution in G.

(2) Prove that intersection of two subgroups of a group is also a subgroup.

(3) Prove that a group of prime order is cyclic.

(4) Let (G, *) be a group then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$, where $a, b \in G$.

(5) If a subgroup H of a group G is a normal subgroups of group $G \; iff \; aha^{-1} \in H \, ; \forall \, a \in G, \forall \, h \in H$.

(6) If H is a normal subgroup of group G with $i_G(H) = m$ then prove that $a^m \in H \, ; \forall \, a \in G$.

(c) Answer any two : **10**

(1) State and prove Cayley's theorem.

(2) Prove that a group cannot be a union of its two proper subgroups.

(3) State and prove Lagrange's theorem for finite groups.

(4) Prove that the combination of two disjoint cycles in S_n is commutative.

(5) Show that $(R_+, \cdot) \cong (R, +)$.
